Sequential Versus Parallel Grammar Formalisms with Respect to Measures of Descriptional Complexity (Algebraic Systems, Formal Languages and Conventional and Unconventional Computation Theory)

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Sequential Versus Parallel Grammar Formalisms with Respect to Measures of Descriptional Complexity

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Abstract

For tabled Lindenmayer systems and their languages, the degree of synchronization and the degree of nondeterminism are well investigated measures of descriptional complexity. In this paper the sequential counterparts of tabled Lindenmayer systems, namely cooperating distributed grammar systems and their pure variant (working in the so-called t-mode of derivation) are treated with respect to these complexity measures. In the pure case, where no distinction between terminal and nonterminal symbols is made, the sequential mechanisms are compared with the parallel ones, investigating whether one mechanism may have a better descriptional complexity than the other one when the same language is described.

1 Introduction

Cooperating distributed grammar systems (CD grammar systems for short) have been introduced in [2] as models of distributed problem solving. They can be considered as a generalization of context-free grammars, where the set of rules is divided into a number of parts each of which is called a component of the system. The components perform derivation steps on a common sentential form taking turns according to some cooperation protocol, the so-called derivation mode.

As E0L systems, that is, extended Lindenmayer systems without interaction, can be viewed as parallel counterparts of context-free grammars, their tabled version (ET0L systems) can be considered as parallel counterparts of CD grammar systems working in the t-mode of derivation. In this derivation mode, a component which became active has to continue the derivation as long as possible, that is, until none of its productions can be applied to the sentential form derived. In what follows, we always assume this t-mode of derivation without further mention.

Analogously, 0L systems or T0L systems may be viewed as parallel counterparts of pure context-free grammars or pure CD grammar systems, respectively, where no distinction between terminal and nonterminal symbols is made. For a more detailed discussion see [1], where also the hierarchical relationships between the language families defined by all systems and their deterministic variants as well as their location within the classical Chomsky hierarchy are investigated.

One reason why such pure grammars and systems are of interest is that there is no distinction between a sentential form and a word in the language generated. Thus, all information about the derivation process is somehow stored in the language. This may be useful for purposes of syntax analysis. Moreover it may help to improve the understanding of the relationship between parallel and sequential rewriting mechanisms.

In the literature on tabled Lindenmayer systems and languages, the degree of synchronization and the degree of nondeterminism are considered as measures of descriptional complexity (e.g., see [6, 7]). For a T0L or ET0L system, the degree of synchronization is simply its number of tables, and the degree of nondeterminism is the maximal number of strings which can be substituted for one symbol according to one table. For a T0L or ET0L language L, these degrees are defined by taking the minimum of the corresponding degrees of the systems generating L.

For CD and pure CD grammar systems these measures of descriptional complexity are defined analogously. The degree of synchronization has been investigated in [2] and [1], respectively, showing that the number of components can be reduced to three in the case of CD grammar systems whereas an infinite hierarchy of language families results in the case of pure CD grammar systems.

The focus of the present paper is twofold. In Section 3 we study the relation between the degree of synchronization and the degree of nondeterminism. First, it is shown that the degree of nondeterminism is bounded by 2 in the case of CD grammar systems. Next, we prove that both degrees are surjective and independent complexity measures with respect to pure CD grammar systems, that is, for any pair (n, r) of positive integers there is a language with degree of nondeterminism n and degree of synchronization r. Finally, we demonstrate that, in the pure case, one can trade off both degrees against each other. Therefore, we consider the complexity measure Compl which is the product of both degrees.

In Section 4 the sequential mechanism of pure CD grammar systems is compared with its parallel counterpart, that of T0L systems, with respect to the complexity measure Compl. Thus we pursue the question of whether one mechanism may have advantages compared to the other one.

2 Definitions and preliminaries

We assume the reader to be familiar with basic notions in the theory of formal languages. With our notation we mainly follow [3]. In general, we have the following conventions: $\subseteq$ denotes inclusion, while $\subset$ denotes strict inclusion. The set of positive integers is denoted by $\mathbb{N}$ and the cardinality of a set M is denoted by $\#M$. By V^+ we denote the set of nonempty words over the alphabet V; if the empty word λ is included, then we use the notation V^*. For $x \in V^*$ and $W \subseteq V$ let $|x|_W$ denote the number of occurrences of letters from W in x. If W is a singleton set $\{a\}$, we simply write $|x|_a$ instead of $|x|_{\{a\}}$.

A *context-free grammar* is a quadruple $G = (N, T, P, S)$, where N and T are disjoint alphabets of nonterminals and terminals, respectively, $S \in N$ is the axiom, and P is a finite set of productions of the form $A \to \alpha$, where $A \in N$ and $\alpha \in (N \cup T)^*$. For x and y in $(N \cup T)^*$, y is derived from x in a direct derivation step according to G, written as $x \Longrightarrow y$, if and only if $x = \gamma_1 A \gamma_2$ and $y = \gamma_1 \alpha \gamma_2$ for some $\gamma_1, \gamma_2 \in (N \cup T)^*$ and $A \to \alpha \in P$. The language generated by G is the set

$$L(G) = \{ w \in T^* \mid S \stackrel{*}{\Longrightarrow} w \},$$

where $\stackrel{*}{\Longrightarrow}$ is the reflexive and transitive closure of $\Longrightarrow$. The family of languages generated by context-free grammars is denoted by $\mathcal{L}(\mathrm{CF})$.

A *pure context-free grammar* (with single axiom) is a triple $G = (V, P, \omega)$, where V is some alphabet, $P \subseteq V \times V^*$ is a finite set of pure context-free productions, and $\omega \in V^+$ is the axiom. The language $L(G)$ is defined in the natural way, that is, as the set of all words over V which can be derived by iterated applications of productions in P, starting at the axiom ω. Note that every sentential form belongs to the language generated.

A *cooperating distributed (CD) grammar system* of degree n, with $n \geq 1$, is an $(n+3)$-tuple

$$G = (N, T, P_1, P_2, \ldots, P_n, S),$$

where, for $1 \leq i \leq n$, (N, T, P_i, S) is a context-free grammar. The production sets $P_1, P_2, \ldots, P_n$ are called *components*. For $1 \leq i \leq n$, let

$$\mathrm{dom}(P_i) = \{\, A \mid A \to \alpha \in P_i \text{ for some } \alpha \,\}.$$

For x, y in $(N \cup T)^*$ and $1 \leq i \leq n$, we write $x \underset{i}{\Longrightarrow} y$ if and only if y is derived from x in a direct derivation step according to (N, T, P_i, S). Hence, subscript i refers to the component to be used. Let $\overset{*}{\underset{i}{\Longrightarrow}}$ denote the reflexive and transitive closure of the relation $\underset{i}{\Longrightarrow}$. In the forthcoming we restrict ourselves to the t-mode of derivation, which is defined as follows: we write $x \overset{t}{\underset{i}{\Longrightarrow}} y$ if and only if $x \overset{*}{\underset{i}{\Longrightarrow}} y$ and there is no z such that $y \underset{i}{\Longrightarrow} z$. The language generated by G is defined as

$$L(G) = \{\, w \in T^* \mid S \overset{t}{\underset{i_1}{\Longrightarrow}} w_1 \overset{t}{\underset{i_2}{\Longrightarrow}} \cdots \overset{t}{\underset{i_m}{\Longrightarrow}} w_m = w \text{ for some } m \geq 0 \text{ and } 1 \leq i_j \leq n \text{ with } 1 \leq j \leq m \,\}.$$

A *pure CD grammar system* (pCD grammar system for short) of degree n is an $(n+2)$-tuple $G = (V, P_1, P_2, \ldots, P_n, \omega)$ such that, for $1 \leq i \leq n$, (V, P_i, ω) is a pure context-free grammar with single axiom. For technical reasons, the t-derivation step according to component P_i is defined as follows: for $x, y \in V^*$, we write $x \overset{t}{\underset{i}{\Longrightarrow}} y$ if and only if one of the following conditions hold:

(i) there exist strings $x_0, x_1, \ldots, x_k$, $k \geq 0$, such that $x_0 = x$, $x_k = y$, $x_j \underset{i}{\Longrightarrow} x_{j+1}$, $0 \leq j \leq k-1$, and there is no z such that $y \underset{i}{\Longrightarrow} z$, or

(ii) $y = x$.

Here $\underset{i}{\Longrightarrow}$ denotes a direct derivation step according to the pure context-free grammar (V, P_i, ω). The set $\mathrm{SF}(x \overset{t}{\underset{i}{\Longrightarrow}} y)$ of the sentential forms of the t-derivation step $x \overset{t}{\underset{i}{\Longrightarrow}} y$ is the set of the strings $\{x_0, x_1, \ldots, x_k\}$.

A t-derivation in a pure CD grammar system is a sequence of t-derivations according to arbitrary components of the system: for $x, y \in V^*$, we write $x \overset{t}{\Longrightarrow} y$ if and only if there are strings $x_0, x_1, \ldots, x_k$, $k \geq 0$, such that $x_0 = x$, $x_k = y$, and $x_j \overset{t}{\underset{i_j}{\Longrightarrow}} x_{j+1}$, for $1 \leq i_j \leq n$, $0 \leq j \leq k-1$. The set $\mathrm{SF}(x \overset{t}{\Longrightarrow} y)$ of its sentential forms is defined to be the union of the sets $\mathrm{SF}(x_j \overset{t}{\underset{i_j}{\Longrightarrow}} x_{j+1})$. The language $L(G)$ generated by a pure CD grammar system G is the set of all sentential forms in a t-derivation in G starting at the axiom ω:

$$L(G) = \{\, w \in V^* \mid w \in \mathrm{SF}(\omega \overset{t}{\Longrightarrow} y) \text{ for some } y \in V^* \,\}.$$

Note that the language consists of all words generated by iterated t-derivation steps and all the intermediate words appearing along these derivations. Sentential forms of derivations where the active component will not terminate are not included, however.

The family of languages generated by (pure) CD grammar systems in t-mode of derivation is denoted by $\mathcal{L}(\mathrm{CD})$ ($\mathcal{L}(\mathrm{pCD})$, respectively).

An ET0L *system* is a quadruple

$$G = (\Sigma, \Delta, H, \omega),$$

where Σ is the total alphabet, $\Delta \subseteq \Sigma$ is the terminal alphabet, H is a finite set of finite substitutions from Σ into Σ^*, and $\omega \in \Sigma^*$ is the axiom. For x and y in Σ^*, we write $x \underset{h}{\Longrightarrow} y$ for some h in H if and only if $y \in h(x)$. A substitution h in H is called a *table*. The language generated by G is defined as

$$L(G) = \{ w \in \Delta^* \mid \omega \underset{h_{i_1}}{\Longrightarrow} w_1 \underset{h_{i_2}}{\Longrightarrow} \cdots \underset{h_{i_m}}{\Longrightarrow} w_m = w \text{ for some } m \geq 0 \text{ and } h_{i_j} \in H \text{ with } 1 \leq j \leq m \}.$$

If $\Delta = \Sigma$, then G is called T0L system and it is written (Σ, H, ω). The family of languages generated by ET0L systems or T0L systems is denoted by $\mathcal{L}(\text{ET0L})$ or $\mathcal{L}(\text{T0L})$, respectively.

Let $G = (\Sigma, \Delta, H, \omega)$ be some ET0L system. The *degree of synchronization* of G, Sync(G) is defined as #H, that is, the number of tables of G. The degree of synchronization of an ET0L language L is given by

$$\text{Sync}_{\text{ET0L}}(L) = \min\{ \text{Sync}(G) \mid G \text{ is a ET0L system with } L(G) = L \}.$$

The *degree of nondeterminism* of G is defined by

$$\text{Det}(G) = \max\{ n_h \mid n_h = \max\{ \#h(a) \mid a \in \Sigma \}, h \in H \}.$$

The degree of nondeterminism of a ET0L language L is given by

$$\text{Det}_{\text{ET0L}}(L) = \min\{ \text{Det}(G) \mid G \text{ is a ET0L system with } L(G) = L \}.$$

The degrees of synchronization and of nondeterminism for T0L systems, CD grammar systems, and pCD grammar systems and their languages are defined analogously. In the case of (pure) CD grammar systems, one simply has to consider components instead of tables. Thus, the degree of synchronization of a (pure) CD grammar system is identical with its number of components which has been called its degree in the above definitions.

Note that an ET0L system or T0L system G with Sync$(G) = 1$ is just an E0L system or 0L system, respectively. Correspondingly, a CD grammar system G with Sync$(G) = 1$ is a context-free grammar. In the pure case, we have a pure context-free grammar only if we disregard the t-mode of derivation.

Furthermore, an L system G with Det$(G) = 1$ is said to be *deterministic* if its only table is a homomorphism rather than a substitution. Analogously, one defines the notion of deterministic (pure) CD grammar systems as considered in [1, 5].

In [1, 2, 6, 7], the following facts has been proved.

Theorem 1

(i) For any ET0L *language* L, $\text{Sync}_{\text{ET0L}}(L) \leq 2$; *there is an* ET0L *language* L *with* $\text{Sync}_{\text{ET0L}}(L) = 2$.

(ii) For any ET0L *language* L, $\text{Det}_{\text{ET0L}}(L) \leq 2$; *there is an* ET0L *language* L *with* $\text{Det}_{\text{ET0L}}(L) = 2$.

(iii) For any language $L \in \mathcal{L}(\text{CD})$, $\text{Sync}_{\text{CD}}(L) \leq 3$; *there is a language* $L \in \mathcal{L}(\text{CD})$ *with* $\text{Sync}_{\text{CD}}(L) = 3$.

(iv) $\mathcal{L}(\mathrm{CD}) = \mathcal{L}(\mathrm{ET0L})$ *and* $L \in \mathcal{L}(\mathrm{CD})$ *is a context-free language if and only if* $\mathrm{Sync}_{\mathrm{CD}}(L) \leq 2$.

(v) *For every pair* (n, r) *of positive integers there is a* T0L *language* $L_{n,r}$ *such that* $\mathrm{Det}_{\mathrm{T0L}}(L_{n,r}) = n$ *and* $\mathrm{Sync}_{\mathrm{T0L}}(L_{n,r}) = r$.

(vi) *For every positive integer* n *there is a language* $L_n \in \mathcal{L}(\mathrm{pCD})$ *with* $\mathrm{Sync}(L_n) = n$.

Results on the degree of nondeterminism for (pure) CD grammar systems have not yet been presented. Moreover its relation to the degree of synchronization is unknown in the sequential case. The subsequent section aims to fill this gap. The only result in this respect is that the degree of nondeterminism cannot be reduced to 1 for both CD grammar systems and their pure variants (see [1, 5]), that is, the deterministic mechanisms are strictly less powerful than the nondeterministic ones.

Theorem 2

(i) *There is a language in* $\mathcal{L}(\mathrm{CD})$ *with* $\mathrm{Det}_{\mathrm{CD}}(L) > 1$.

(ii) *For every positive integer* n *there is a language* $L \in \mathcal{L}(\mathrm{pCD})$ *with* $\mathrm{Sync}_{\mathrm{pCD}}(L_n) = n$ *and* $\mathrm{Det}_{\mathrm{pCD}}(L_n) > 1$.

In the following section it is shown that, for some pCD languages, one can trade the degree of nondeterminism for the degree of synchronization. This is achieved using the complexity measure Compl defined for T0L and pCD grammar systems G as

$$\mathrm{Compl}(G) = \mathrm{Det}(G) \cdot \mathrm{Sync}(G)\,.$$

This measure has been considered in [4] for k-limited T0L systems. This measure depends on the total number of productions of G and, therefore, reflects its complexity well if the size of the alphabet is fixed. Let $X \in \{\mathrm{T0L}, \mathrm{pCD}\}$. For a language $L \in \mathcal{L}(X)$, this measure is defined to be

$$\mathrm{Compl}_X(L) = \min\{\, \mathrm{Compl}(G) \mid G \text{ is of type } X \text{ and } L(G) = L\,\}\,.$$

By definition, we have

$$\mathrm{Det}_X(L) \cdot \mathrm{Sync}_X(L) \leq \mathrm{Compl}_X(L)\,.$$

3 On the complexity of CD grammar systems and their pure variants

In this section we first show that one can reduce the degree of nondeterminism of a CD grammar system G to 2.

Theorem 3 *There is an algorithm which constructs, for any* CD *grammar system* G, *an equivalent* CD *grammar system* G' *with* $\mathrm{Det}(G') = 2$.

Proof. Let $G = (N, T, P_1, P_2, \ldots, P_k, S)$ be a CD grammar system with $\mathrm{Det}(G) = n$, $n \geq 2$.

Construct $G' = (N', T, P_1', P_2', \ldots, P_k', S)$ from G as follows. The axiom remains the same. We introduce new nonterminal symbols, such that

$$N' = N \cup \{\, A_i \mid A \in N,\ 1 \leq i \leq n\,\}.$$

For $1 \leq i \leq k$, P_i' is defined as the union

$$\bigcup_{A \in \mathrm{dom}(P_i)} P_{i,A}\,,$$

where $P_{i,A}$ is constructed in the following way. Let $\{\, v \mid A \to v \in P_i \,\} = \{v_1, v_2, \ldots, v_m\}$, $m \leq n$.

If $m \leq 2$, then

$$P_{i,A} = \{\, A \to v \mid A \to v \in P_i \,\}.$$

Otherwise we set

$$P_{i,A} = \{A \to A_1\} \cup \{\, A_j \to A_{j+1} \mid 1 \leq j \leq m-1 \,\} \cup \{\, A_j \to v_j \mid 1 \leq j \leq m \,\}.$$

Then $\mathrm{Det}(G') = 2$.

Both grammars G' and G generate the same language in the t-mode, that is, $L(G') = L(G)$, which can be seen from the fact that the t-derivations in G can be simulated in G' by the newly introduced nonterminal symbols. Except these simulating derivations no others derivations can be obtained. □

Together with Theorem 2 (i), we have the following Corollary.

Corollary 4 *For any language $L \in \mathcal{L}(\mathrm{CD})$, we have $\mathrm{Det}_{\mathrm{CD}}(L) \leq 2$, and there is a language $L \in \mathcal{L}(\mathrm{CD})$ with $\mathrm{Det}_{\mathrm{CD}}(L) = 2$.* □

According to the constructions in the proofs of Theorem 1 (iii), see [2], and Theorem 3 above, for any CD grammar system G one can reduce the degree of synchronization to 3 and the degree of nondeterminism to 2 independently from each other.

Next, we consider the pure case. We are going to show that both the degree of nondeterminism and the degree of synchronization are surjective measures of descriptional complexity, that is, they give rise to an infinite hierarchy of families of languages. Moreover, they are independent measures.

Theorem 5 *For every pair of integers $(n, r) \in \mathbb{N}^2$ there exists a language $L_{n,r} \in \mathcal{L}(\mathrm{pCD})$ with $\mathrm{Det}_{\mathrm{pCD}}(L_{n,r}) = n$ and $\mathrm{Sync}_{\mathrm{pCD}}(L_{n,r}) = r$.*

The proof is omitted here due to lack of space.

Next, we show that there are languages with respect to which one can trade the degree of nondeterminism with the degree of synchronization or vice versa. A similar effect has been observed in [6] for T0L languages.

Theorem 6 *For any integer $n \geq 1$, there exists a language $L_n \in \mathcal{L}(\mathrm{pCD})$ with*
$\mathrm{Det}_{\mathrm{pCD}}(L_n) \cdot \mathrm{Sync}_{\mathrm{pCD}}(L_n) = 1$ *and* $\mathrm{Compl}_{\mathrm{pCD}}(L_n) = n$.

The proof is left out here because of space reasons.

4 T0L versus pure CD grammar systems

In this section the sequential mechanism of pure CD grammar systems is compared with its parallel counterpart, the T0L systems, investigating whether there are languages which can be described with one mechanism type more economically than with the other type. Here, the economy is measured by the complexity measure Compl.

First, it is shown that there is an infinite sequence of languages where parallel mechanisms turn out to be much more efficient than the sequential ones. In fact, the measure Compl for pCD grammar systems is growing asymptotically faster than Compl for T0L systems.

Theorem 7 *For any positive integer n, there exists a language M_n, such that $M_n \in \mathcal{L}(\text{pCD}) \cap \mathcal{L}(\text{T0L})$ and*

$$\lim_{n\to\infty} \frac{\text{Compl}_{\text{T0L}}(M_n)}{\text{Compl}_{\text{pCD}}(M_n)} = 0.$$

Proof: For $n \geq 1$, consider the finite language

$$M_n = \{ (bc)^{n+1}, d\} \cup \{b^{j_1}a^{n+2}cb^{j_2}a^{n+2}c \ldots b^{j_{n+1}}a^{n+2}c \mid 2 \leq j_1, \ldots, j_{n+1} \leq n+1 \}.$$

For $n \geq 1$, the language M_n is generated by the pCD grammar system $G_n = (\{a, b, c, d\}, P_1, d)$ where

$$P_1 = \{ d \to w \mid w \in M_n, w \neq d \}.$$

Since there are $K = n^{n+1} + 1$ words in M_n which are different from d, there are exactly K productions in the single component P_1 of G_n. Hence, $\text{Compl}(M_n) \leq K$.

On the other hand, all these K productions are needed in order to generate M_n by some pure CD grammar system. This is seen as follows. Let H be a pCD grammar system with $L(H) = M_n$.

Using analogous arguments as in the proof of Theorem 5 it is shown that, except from productions of the form $x \to x$, only productions with the letter d on the left-hand sides can be applied in terminating derivations in H. Hence, $\omega = d$ has to hold.

Furthermore, it follows that the axiom d is the only word in M_n which can derive other words in M_n. Thus, all the K productions in P_1 above are needed in H. Since a distribution of these productions to different components does not affect the size of $\text{Det}(H) \cdot \text{Sync}(H)$, it follows that $\text{Compl}_{\text{pCD}}(M_n) = K = n^{n+1} + 1$.

Next, for $n \geq 1$, we present a T0L system generating M_n. Clearly, this is done with the system $\Gamma_n = (\{a, b, c, d\}, h, d)$ where the substitution h is defined by

$$h(a) = h(b) = \lambda,\ h(c) = \{ b^i a^{n+2} c \mid 2 \leq i \leq n+1 \},\ \text{and } h(d) = (bc)^{n+1}.$$

Therefore[1], we have $\text{Compl}_{\text{T0L}}(M_n) \leq n$. In conclusion,

$$\lim_{n\to\infty} \frac{\text{Compl}_{\text{T0L}}(M_n)}{\text{Compl}_{\text{pCD}}(M_n)} = \lim_{n\to\infty} \frac{o(n)}{n^{n+1}+1} = 0.$$ □

It turns out that there is also an infinite sequence of languages for which the sequential mechanism is more economical, although the advantage is not of the same degree as presented in Theorem 7.

Theorem 8 *For any positive integer $n \geq 1$ there is a language K_n such that $K_n \in \mathcal{L}(\text{pCD}) \cap \mathcal{L}(\text{T0L})$ and $\text{Compl}_{\text{T0L}}(L_n) - \text{Compl}_{\text{pCD}}(L_n) = n$.*

Proof. For any positive integer, consider the language

$$K_n = \{a^2\} \cup \{ acb^ic \mid 1 \leq i \leq n \} \cup \{ cb^ica \mid 1 \leq i \leq n \} \cup \{ (cb^ic)^2 \mid 1 \leq i \leq n \}.$$

Obviously, K_n is generated by the pCD grammar

$$G_n = (\{a, b, c\}, P_1, P_2, \ldots, P_n, a^2)$$

[1]A similar language has been considered in [6] in the proof of the statement given in Theorem 1(v). From the arguments in that proof we know that $\text{Compl}_{\text{T0L}}(M_n) = n$ holds.

with $P_i = \{a \to cb^i c\}$, $1 \le i \le n$.

Using arguments as in the proofs above, one shows that all productions are needed in order to generate K_n with a pCD grammar system and, moreover, that they need to be contained as single productions in separate components. Hence, $\mathrm{Compl}_{\mathrm{pCD}}(K_n) = n$.

On the other hand, L_n is generated by the T0L system

$$\Gamma = (\{a, b, c\}, h_1, h_2, \ldots, h_n, a^2),$$

where, for $1 \le i \le n$, $h_i(a) = \{a, cb^i c\}$, $h(b) = b$, and $h(c) = c$.

We can simulate sequential rewriting by introducing productions of the form $x \to x$. As in the sequential case, one verifies that this is the only possibility to generate K_n with a T0L system. Since the supplementals of the tables increase the degree of nondeterminism to two, it follows that $\mathrm{Compl}_{\mathrm{T0L}}(K_n) = 2n$. □

5 Concluding remarks

The degree of synchronization and the degree of nondeterminism are well-investigated measures of descriptional complexity in the theory of tabled Lindenmayer systems. In this paper, these measures are considered with respect to cooperating distributed grammar systems and their pure variants which can be viewed as sequential counterparts of ET0L and T0L system, respectively. The results proved correspond to analogous ones known for L systems, but do not exists for CD grammar systems.

Moreover, the pure sequential mechanisms are compared with their parallel analogues with respect to the complexity measure Compl, the product of the degrees of synchronization and nondeterminism. It is shown that there are T0L systems more succinct than pure CD grammar systems and, vice versa, there are examples of the sequential mechanism being more economical. More precisely, there is an infinite sequence of languages for which the measure $\mathrm{Compl}_{\mathrm{pCD}}$ is growing asymptotically faster than $\mathrm{Compl}_{\mathrm{T0L}}$. The converse result is obtained with the help of an infinite sequence of languages for which both $\mathrm{Compl}_{\mathrm{pCD}}$ and $\mathrm{Compl}_{\mathrm{T0L}}$ are growing as linear functions. Nevertheless, it is shown that the difference between these measures can be arbitrarily large. One could have the impression that this result cannot be improved since any sequential mechanism can be simulated by a parallel one when just the productions for all x in alphabet, $x \to x$ are added to each component of the given pure CD grammar systems. On the other hand, one has to be aware of the facts that only pCD grammar systems are treated which are generating T0L languages and that the resulting T0L systems have to rule out all derivations which are, in principle, possible but not eventually terminating. Whether or not this is possible is left open here.

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